Comment on "Topological Transitions in Berry's Phase Interference Effects"

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The paper by Lyanda-Geller (henceforth LG) [1] predicts a variation from π to zero of Berry's phase, which may manifest itself in a step-like current-magnetic field and current-gate voltage characteristics predicted for in-plane magnetoresistance of rings in noncentrosymmetric materials.

As a demonstrating example LG considers a spin 1/2 evolving according to the following Schrödinger equation

$$\frac{d}{dt}\left|\psi\right\rangle = i\mathbf{\Omega}\cdot\sigma\left|\psi\right\rangle , \tag{1}$$

where σ is Pauli matrix vector, and the vector $\mathbf{\Omega}$, which lies in the xy plane, is given by

$$\mathbf{\Omega} = \left(\omega_1 - \omega_0\cos\left(\omega t\right), \omega_0\sin\left(\omega t\right), 0\right), \tag{2}$$

where ω, ω_0, ω_1 are constants.

The state at the initial time $t_0 = -\pi/\omega$ is assumed to be an eigenstate of the instantaneous Hamiltonian $\mathcal{H}\left(t_0\right)$. In general, if $\omega_1 \neq \omega_0$ and ω is sufficiently small the system is expected to evolve adiabatically, namely, at any later time $t > t_0$ the state $\left|\psi\left(t\right)\right\rangle$ will remain approximately an eigenvector of the instantaneous Hamiltonian $\mathcal{H}\left(t\right)$. However, LG further claims that adiabatic evolution is possible also for the case $\omega_1 = \omega_0$ provided that ω is sufficiently small. Consequently, LG concludes that the conductance may exhibit an abrupt jump as the parameter ω_1 is varied across the point $\omega_1 = \omega_0$.

In this comment we claim that, contrary to Ref. [1], the conductance steps predicted by LG are not abrupt but rather they occur along a finite range. In general, such abrupt jumps are ruled out since the system under consideration has a linear respond [2]. For a finite time interval, the change in the final state of the system cannot remain finite in the limit where the perturbation causing the change (modifying ω_1) approaches zero. This implies that the change in conductance occurs along a finite range.

To probability $P_{+-}^{(t_1)}$ in Eq. (8) of Ref. [1] is calculated correctly. Indeed, for the case $\omega_1 = \omega_0$, the state of the system will remain nearly unchanged as the curve $\mathbf{\Omega}\left(t\right)$ crosses the degeneracy point at the origin provided that ω is sufficiently small. However, across this point the local eigenvectors $\left|n_\pm\right\rangle$ change abruptly, namely, $\left|n_+\right\rangle$ becomes $\left|n_-\right\rangle$ and vise versa. Therefore, $P_{+-}^{(t_1)}$ in Ref. [1] is not the probability to have a state mixing (Zener transition), but rather the probability *not* to have one.

To further support this conclusion we integrate Eq. 1 numerically from $t_0 = -\pi/\omega$ to $t_1 = \pi/\omega$ [3]. For the example depicted in Fig. 1 below we chose $\omega_0/\omega = 1000$. In Fig. 1 (a) the vector $\mathbf{\Omega}\left(t\right)$ is depicted for the case $\Delta \equiv \left(\omega_1 - \omega_0\right)/\omega_0 = 0.0015$, and in Fig. 1 (b) the polarization vector $\left\langle\psi\right|\sigma\left|\psi\right\rangle$ is seen for the same value of Δ. For this example the state at the final time t_1 is nearly orthogonal to the initial state at time t_0, indicating that a state mixing (Zener transition) occurs. However for larger values of $|\Delta|$, adiabaticity is restored, as can be seen in Fig. 1 (c), where the numerically calculated probability of Zener transition is plotted as a function of Δ. The same plot also shows the Berry's phase γ_B plotted vs. Δ. Near $\Delta = 0$ indeed γ_B changes by π as predicted by LG, however, this occurs along a finite range of Δ rather than abruptly.

[1] Y. Lyanda - Geller, Phys. Rev. Lett. **71**, 657 (1993).
[2] Eyal Buks, arXiv: quant-ph/0510119.
[3] Eyal Buks, J. Opt. Soc. Am. B (to be published), arXiv: quant-ph/0412198.

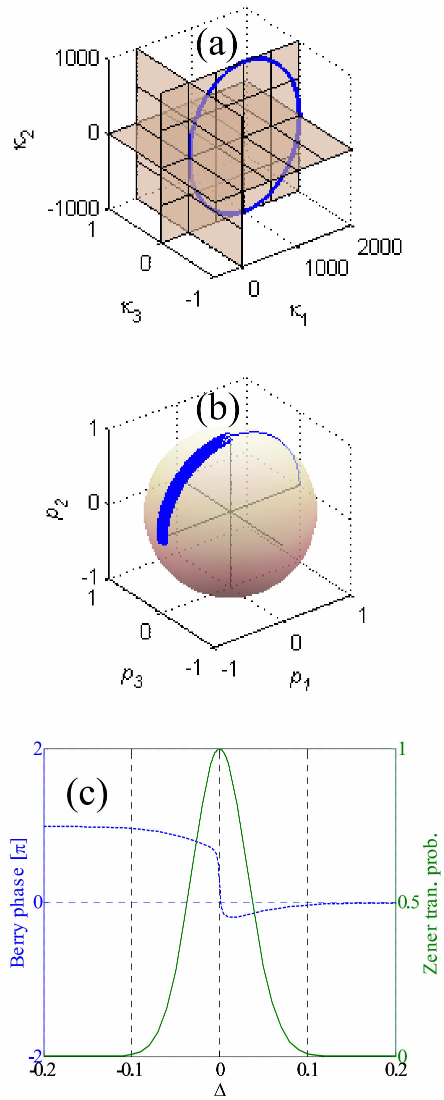

FIG. 1: Numerical integration of Eq. 1.